

66[G].—KENNETH HOFFMAN & RAY KUNZE, *Linear Algebra*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1961, ix + 332 p., 24 cm. Price \$7.50.

This book was written to provide a text for the undergraduate course in linear algebra at the Massachusetts Institute of Technology. It covers vector spaces, linear equations and transformations, polynomials, determinants, invariant direct-sum decompositions, the rational and Jordan canonical forms for matrices, inner product spaces, and bilinear forms. The treatment is imbued with the modern axiomatic and abstract spirit but many concrete illustrative examples and exercises are given so that a serious, persevering student can master it. When he has done so, he is well equipped to study not only applied mathematics and theoretical physics but also mathematical analysis.

We heartily recommend the book either as a text or for private study. The authors are evidently teachers of experience and good judgment who know and like their subject. Their aim is high, and only the best students will master the course, but all the students will be the better for the part of it they learn. In these difficult days for students it is well to have texts so well planned and written as this one.

F. D. MURNAGHAN

Applied Mathematics Laboratory  
David Taylor Model Basin  
Washington 7, D. C.

67[G].—G. TEMPLE, *Cartesian Tensors*, John Wiley & Sons, Inc., New York, 1961, vii + 92 p., 19 cm. Price \$2.75.

This introduction to vector and tensor algebra is well planned and should prove of value to the better than average student. The amount of material covered in less than 100 pages is surprising; in addition to the usual topics, there are chapters on isotropic tensors, spinors, and orthogonal curvilinear tensors. The influence of Weyl is evident in the treatment of isotropic tensors and of Brauer and Weyl in the treatment of spinors. We heartily recommend the book, which is addressed to first year students "pursuing an Honours course in Mathematics or Physics" in England. We found only two points where our instruction of the subject would differ slightly.

First, in the reduction of a symmetric tensor to diagonal form the author discusses the ratio  $f(u) = \frac{S_{\alpha\beta}u_\alpha u_\beta}{u_\alpha u_\alpha}$  and states that this must assume its lower bound when  $\partial f/\partial u_\alpha = 0$ . This assumes, tacitly, that the lower bound is not assumed on the boundary of the region  $-1 \leq u_\alpha \leq 1$ . The author's statement that one must consider separately "the cases in which the matrices of  $S - \lambda_i u$  are of ranks 3, 2 or 1" does not apply to Schur's induction method, which is indifferent to possible equalities of the characteristic numbers of  $S$ .

Second, in the treatment of spinors I would emphasize more the two-valued nature of the correspondence between rotations,  $R$ , and unimodular 2-dimensional unitary matrices,  $U$ . Thus, while to each  $U$  there corresponds only one  $R$ , to each  $R$  there correspond the two matrices  $\pm U$ . For me, the displacement, in modern physics, of vectors from their front rank position, in favor of spinors, is due to the fact that the 2-dimensional unimodular unitary group is not a proper representa-

tion, but rather an ambivalent one, of the 3-dimensional rotation group. Thus, there exist tensors of the unitary group which are not found amongst the tensors of the rotation group, and these tensors, the so-called spin-tensors of the rotation group, have their physical significance; on the other hand, all tensors of the rotation group are found among the tensors of the unitary group.

F. D. MURNAGHAN

68 [K].—E. D. BARRACLOUGH & E. S. PAGE, "Tables for Wald tests for the mean of a normal distribution," *Biometrika*, v. 46, 1959, p. 169–177.

Let  $x$  be normally distributed with unknown mean  $\theta$  and known variance  $\sigma^2$ . The Wald sequential test for  $\theta = \theta_0$  against the alternative  $\theta = \theta_1$  ( $\theta_1 > \theta_0$ ) consists in taking observation  $x_{n+1}$  as long as  $a < (\theta_1 - \theta_0) / \sum_{i=1}^n x_i / \sigma^2 + n(\theta_0^2 - \theta_1^2) / 2\sigma^2 < b$ ; sampling stopping when this relation first fails, with  $\theta = \theta_0$  accepted if the left-hand inequality fails and  $\theta = \theta_1$  accepted if the right-hand inequality fails. Let  $Z = -a\sigma / (\theta_1 - \theta_0)$ ,  $h = (b - a)\sigma / (\theta_1 - \theta_0)$ ,  $P_-$  = probability of accepting  $\theta = \theta_0$  when true,  $P_+$  = probability of accepting  $\theta = \theta_0$  when  $\theta = \theta_1$ ,  $N_-$  = average sample number when  $\theta = \theta_0$ ,  $N_+$  = average sample number when  $\theta = \theta_1$ , and  $\mu = (\theta_1 - \theta_0) / 2$ . Table 1 of the Appendix contains 2D values of  $h$  and  $Z$  for  $P_+ = .05, .10(.10) .70, P_- = .95, .99, .995, .999$ , and  $\mu = .25(.25)1.00$ . The values of  $a$  and  $b$  are determined by  $h$  and  $z$ . For Table 2 of the Appendix (the same combination of values for  $P_+, P_-, \mu$  occur as for Table 1), 2D values are given for  $N_+$  and  $N_-$ . Charts I–IV of the Appendix contain curves of  $P_+$  and  $P_-$  as functions of  $h$  and  $Z$  for given  $\mu$ . These charts are obtained directly from Table 1 and can be used to determine the operating characteristics of the test for given  $a, b, \sigma, \theta_0$ , and  $\theta_1$ . Also, a comparison is made between Wald's approximations (to the operating characteristics and the average sample numbers) and the true values, for ten combinations of values for  $h, Z, \mu$  (Text-Table 1). The conclusion reached is that Wald's approximations are not acceptably accurate for many applications when  $\mu \geq .25$ .

J. E. WALSH

System Development Corporation  
Santa Monica, California

69 [K].—A. T. BHARUCHA-REID, *Elements of the Theory of Markov Processes and their Applications*, McGraw-Hill Book Co., Inc., New York, 1960, xi + 468 p., 24 cm. Price \$11.50.

In recent years the notions of probability have become increasingly important in the building of models of the world around us. This is true, for example, in certain of the physical sciences, social sciences, and in the simulation of military and other operations. Sometimes probabilistic notions appear directly as basic ingredients of the model, sometimes indirectly as the result of applying Monte Carlo methods to the solution of certain types of functional equations.

The mathematical abstraction of an empirical process whose development is governed by probabilistic laws is known as a stochastic process. A special class of these processes are Markov processes in which the development subsequent to a time  $t$  depends (probabilistically) only upon the state of the process at  $t$  and not